ENG1091

Mathematics for Engineering: Algebra

Monash University, Australia

1 Vectors in 3D

Length of a vector

$$|\mathbf{v}| = \sqrt{x^2 + y^2 + z^2}$$

Dot Product

$$\mathbf{v} \cdot \mathbf{w} = v_x w_x + v_y w_y + v_z w_z = |\mathbf{v}||\mathbf{w}| cos\theta$$

Cross Product

$$\mathbf{v} \times \mathbf{w} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ v_x & v_y & v_z \\ w_x & w_y & w_z \end{vmatrix}$$

Scalar projection $\mathbf{v}$ in direction of $\mathbf{w}$

$$v_w = \frac{\mathbf{v} \cdot \mathbf{w}}{|\mathbf{w}|}$$

Vector projection $\mathbf{v}$ in direction of $\mathbf{w}$

$$v_w = \left(\frac{\mathbf{v} \cdot \mathbf{w}}{|\mathbf{w}|^2} \right) \mathbf{w}$$

2 Lines in 3D

Parametric Equation of a Line

$$\begin{Bmatrix} x(t) = a + pt \\ y(t) = b + qt \\ z(t) = c + rt \end{Bmatrix}$$

Symmetric Form of Line Equation

$$\frac{x-a}{p} = \frac{y-b}{q} = \frac{z-c}{r}$$

Vector Equation of a Line

$$\mathbf{r}(t) = \mathbf{d} + t\mathbf{v}$$

3 Planes in 3D

Cartesian Equation of a Plane

$$ax + by + cz = d$$

Parametric Equation of a Plane

$$\begin{Bmatrix} x(u,v) = a + pu + lv \\ y(u,v) = b + qu + mv \\ z(u,v) = c + ru + nv \end{Bmatrix}$$

Vector Equation of a Plane

$$\mathbf{n} \cdot (\mathbf{r} - \mathbf{d}) = 0$$

4 Matrices

Matrix Multiplication

$$\mathbf{AB} = \begin{pmatrix} a & b & c \\ x & y & z \end{pmatrix} \begin{pmatrix} \alpha & \rho \\ \beta & \sigma \\ \gamma & \tau \end{pmatrix} = \begin{pmatrix} a\alpha + b\beta + c\gamma & a\rho + b\sigma + c\tau \\ x\alpha + y\beta + z\gamma & x\rho + y\sigma + z\tau \end{pmatrix}$$

Transpose

$$\begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix}^{\mathrm{T}} = \begin{bmatrix} a & c & e \\ b & d & f \end{bmatrix}$$

Identity Matrix

$$I_n = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix}$$

Symmetric and Skew Symmetric Matrix

- Symmetric: $\mathbf{A} = \mathbf{A}^{\mathbf{T}}$
- Skew-Symmetric: $\mathbf{A} = -\mathbf{A}^{\mathbf{T}}$

Basic Properties

- $AB \neq BA$
- $(AB)C = A(BC)$
- $(A^T)^T = A$
- $(AB)^T = B^T A^T$

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