MATH1051

Calculus & Linear Algebra

University of Queensland

1 Complex Numbers

$$z = a + ib$$

Where:

$$i^2 = -1$$

Modulus

$$|z| = \sqrt{a^2 + b^2}$$

Conjugate

If

$$z = a + ib$$

then

$$\overline{z} = z - ib$$

Properties of Modulus and Conjugate

$$|zw| = |z||w|$$

$$\left|\frac{z}{w}\right| \frac{|z|}{|w|}$$

$$|z + w| \leq |z| + |w|$$

$$|z - w| \geq |z| - |w|$$

$$\overline{z \pm w} = \overline{z} \pm \overline{w}$$

$$\overline{zw} = \overline{zw}$$

Polar Form

$$z = rcis\theta = r(\cos\theta + i\sin\theta)$$

Where r is the modulus $||z||$ and θ the argument $\arg z$.

Polar Form Operations and Properties

$$rcis(\theta) \times pcis(\phi) = rpcis(\theta + \phi)$$

$$\frac{rcis(\theta)}{pcis(\phi)} = \frac{r}{p}cis(\theta - \phi)$$

$$\arg(zw) = \arg(z) + \arg(w)$$

$$\arg\left(\frac{z}{w}\right) = \arg(z) - \arg(w)$$

Euler's Formula

$$e^{ix} = \cos x + i\sin x$$

Where e is Euler's Number and x is real.

De Moivre's Theorem

Given $z = re^{i\theta}$:

$$z^n = r^n e^{in\theta}$$

2 Vectors

Length of a vector

$$|\mathbf{v}| = \sqrt{x^2 + y^2 + z^2}$$

Dot Product

$$\mathbf{v} \cdot \mathbf{w} = v_x w_x + v_y w_y + v_z w_z = |\mathbf{v}||\mathbf{w}|cos\theta$$

Angle Between Vectors

$$\cos\theta = \frac{\mathbf{a} \cdot \mathbf{b}}{\|\mathbf{a}\|\,\|\mathbf{b}\|}$$

Cross Product

$$\mathbf{v} \times \mathbf{w} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ v_x & v_y & v_z \\ w_x & w_y & w_z \end{vmatrix}$$

Area of a Triangle

Formed by vectors $\mathbf{v}$ and $\mathbf{u}$ is:

$$A = \frac{|v \times u|}{2}$$

Scalar projection $\mathbf{v}$ in direction of $\mathbf{w}$

$$v_w = \frac{\mathbf{v} \cdot \mathbf{w}}{|\mathbf{w}|}$$

Vector projection $\mathbf{v}$ in direction of $\mathbf{w}$

$$v_w = \left(\frac{\mathbf{v} \cdot \mathbf{w}}{|\mathbf{w}|^2}\right)\mathbf{w}$$

Triangle Inequality

$$|\mathbf{v} + \mathbf{w}| \leq |\mathbf{v}| + |\mathbf{w}|$$

CHEAT SHEET

spoonfeedme

Sign up to get this **cheatsheet completely free** as well as **view more videos and notes.**